

A Relativistic Symmetry in Nuclei: Its origins and consequences

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We review the status of quasi-degenerate doublets in nuclei, called pseudospin doublets, which were discovered about thirty years ago and the origins of which have remained a mystery, until recently. We show that pseudospin doublets originate from an $SU(2)$ symmetry of the Dirac Hamiltonian which occurs when the sum of the scalar and vector potentials is a constant. Furthermore, we survey the evidence that pseudospin symmetry is approximately conserved in nuclear spectra and eigenfunctions and in nucleon-nucleus scattering for a Dirac Hamiltonian with realistic nuclear scalar and vector potentials.

1. INTRODUCTION

I first met Achim Richter at an international nuclear physics conference in Crete in 1982. We talked about the "scissors" mode, an excited state with quantum numbers $J^\pi = 1^+$, which he and his collaborators had discovered experimentally at 3.075 MeV in ^{156}Gd [1]. My collaborators and I had predicted in a schematic shell model calculation a 1^+ in the nearby Samarium isotopes at 3.0 - 3.4 MeV (depending on the isotope) [2]. No one paid any attention to that calculation. I told Achim about our calculation at the meeting, and, to his credit, he did acknowledge our work although everybody else continued to ignore our prediction. Years later during a four month sabbatical to the Max Planck Institute in Heidelberg on a Humboldt Senior Scientist Award, I visited the Technical University at Darmstadt for one day. I was very impressed with the facility and the group that Achim was leading. He and his colleagues told me about the interesting work they were doing which included by then a very exhaustive study of the "scissors" mode and its excitation strength in many nuclei. The visit must have been very inspirational because the next day after returning to Heidelberg I derived (within the interacting boson model [3]) a sum rule which related the total "scissors" excitation strength to the number of quadrupole pairs of nucleons in the ground state [4]. Later on Achim, P. von Neumann-Cosel, H. Bauer, and I collaborated on a paper which used this sum rule to correlate the "scissors" mode excitation strength to the nuclear deformation [5]. My work on the "scissors mode" was very exciting for me and I am glad that Achim persisted and found that little 1^+ state.

In this paper I shall talk about quasi-degenerate doublets discovered more than thirty years ago [6,7] called pseudospin symmetry doublets. One of the results I shall show is that this symmetry is related to another interest of Achim, namely the measurement of " ℓ " forbidden magnetic dipole transitions [8].

2. PSEUDOSPIN SYMMETRY

The spherical shell model orbitals that were observed to be quasi-degenerate have non-relativistic quantum numbers $(n_r, \ell, j = \ell + 1/2)$ and $(n_r - 1, \ell + 2, j' = j + 1 = \ell + 3/2)$ where n_r , ℓ , and j are the single-nucleon radial, orbital, and total angular momentum quantum numbers, respectively [6,7]. This doublet structure is expressed in terms of a “pseudo” orbital angular momentum $\tilde{\ell} = \ell + 1$, the average of the orbital angular momentum of the two states in doublet, and “pseudo” spin, $\tilde{s} = 1/2$. For example, $(n_r s_{1/2}, (n_r - 1) d_{3/2})$ will have $\tilde{\ell} = 1$, $(n_r p_{3/2}, (n_r - 1) f_{5/2})$ will have $\tilde{\ell} = 2$, etc. Since $j = \tilde{\ell} - \frac{1}{2}$ and $j' = \tilde{\ell} + \frac{1}{2}$, the energy of the two states in the doublet are then approximately independent of the orientation of the pseudospin. Some examples are given in Table 1.

Table 1

Experimental (Exp) and relativistic mean field (RMF) [9] pseudospin binding energy splittings $\epsilon_{j'=\tilde{\ell}+1/2} - \epsilon_{j=\tilde{\ell}-1/2}$ for various doublets in ^{208}Pb .

$\tilde{\ell}$	$(n_r - 1, \ell + 2, j' = \ell + 3/2)$ $-(n_r, \ell, j = \ell + 1/2)$	$\epsilon_{j'=\tilde{\ell}+1/2} - \epsilon_{j=\tilde{\ell}-1/2}$ (Exp) (MeV)	$\epsilon_{j'=\tilde{\ell}+1/2} - \epsilon_{j=\tilde{\ell}-1/2}$ (RMF) (MeV)
4	$0h_{9/2} - 1f_{7/2}$	1.073	2.575
3	$0g_{7/2} - 1d_{5/2}$	1.791	4.333
2	$1f_{5/2} - 2p_{3/2}$	-0.328	0.697
1	$1d_{3/2} - 2s_{1/2}$	0.351	1.247

Pseudospin “symmetry” was shown to exist in deformed nuclei as well [10] and has been used to explain features of deformed nuclei, including superdeformation and identical bands [11–14], and example of which is given in Fig. 1.

However, the origin of pseudospin symmetry remained a mystery and “no deeper understanding of the origin of these (approximate) degeneracies” existed [15]. In this paper we shall review more recent developments that show that pseudospin symmetry is a relativistic symmetry [16–18].

3. SYMMETRIES OF THE DIRAC HAMILTONIAN

The success of the shell model implies that nucleons move in a mean field produced by the interactions between the nucleons. Normally, it suffices to use the Schrodinger equation to describe the motion of the nucleons in this mean field. However, in order to understand the origin of pseudospin symmetry, we need to take into account the motion of the nucleons in a relativistic mean field and thus use the Dirac equation. The Dirac

Figure 1. Two quasi-degenerate rotational bands in ^{187}Os , one with pseudospin unaligned with the body-fixed pseudo-angular momentum projection $\tilde{\Lambda} = 1$ and one aligned [13,14].

Hamiltonian, H , with an external scalar, V_S , and vector, V_V , potentials is given by:

$$H = \vec{\alpha} \cdot \vec{p} + \beta(M + V_S) + V_V, \quad (1)$$

where we have set $\hbar = c = 1$, $\vec{\alpha}$, β are the usual Dirac matrices, M is the nucleon mass, and \vec{p} is the three momentum. The Dirac Hamiltonian is invariant under an $\text{SU}(2)$ algebra for two limits: $V_S - V_V = \text{constant}$ and $V_S + V_V = \text{constant}$ [19]. The first condition leads to a spin symmetry which is relevant for mesons [19,20] while the second leads to pseudospin symmetry [17]. The generators for the $\text{SU}(2)$ pseudospin symmetry, $\hat{\tilde{S}}_i$, which commute with the Dirac Hamiltonian, $[H, \hat{\tilde{S}}_i] = 0$, are given by $\hat{\tilde{S}}_i = \begin{pmatrix} \hat{s}_i & 0 \\ 0 & \hat{s}_i \end{pmatrix}$ where \hat{s}_i is the usual spin operator, $\hat{\tilde{s}}_i = U_p \hat{s}_i U_p$ and $U_p = 2\frac{\hat{s} \cdot \vec{p}}{p}$ is a unitary helicity operator [21]. These pseudospin generators have the spin operator \hat{s}_i operating on the lower component of the Dirac wave function which has the consequence that the spatial wavefunctions for the two states in the pseudospin doublet are identical to within an overall phase.

This symmetry for $V_S + V_V = \text{constant}$ is general and applies to spherical, axially deformed, triaxial, gamma unstable, etc., nuclei. In the case for which the potentials are spherically symmetric, the Dirac Hamiltonian conserves the pseudo-orbital angular momentum, the generators of which are, $\hat{\tilde{L}}_i = \begin{pmatrix} \hat{\tilde{\ell}}_i & 0 \\ 0 & \hat{\ell}_i \end{pmatrix}$, where $\hat{\tilde{\ell}}_i = U_p \hat{\ell}_i U_p$, $\hat{\ell}_i = \vec{r} \times \vec{p}$. Since U_p conserves the total angular momentum but \vec{p} changes the orbital angular momentum by one unit because of parity conservation, if the lower component of the Dirac wave

function orbital angular momentum is $\tilde{\ell}$, the upper component also has total angular momentum j , but orbital angular momentum $\ell = \tilde{\ell} \pm 1$. If $j = \tilde{\ell} + 1/2$, then it follows that $\ell = \tilde{\ell} + 1$, whereas if $j = \tilde{\ell} - 1/2$, then $\ell = \tilde{\ell} - 1$. This agrees with the pseudospin doublets originally observed [6,7] and discussed at the beginning of this paper. This relativistic interpretation also gives the physical significance of the pseudo-orbital angular momentum $\tilde{\ell}$ as the “orbital angular momentum” of the lower component.

For axially symmetric deformed nuclei, there is a U(1) generator corresponding to the pseudo-orbital angular momentum projection along the body-fixed symmetry axis which is conserved in addition to the pseudospin, $\hat{\lambda} = \begin{pmatrix} \hat{\Lambda} & 0 \\ 0 & \hat{\Lambda} \end{pmatrix}$, where $\hat{\Lambda} = U_p \hat{\Lambda} U_p$.

In general, the eigenfunctions of the Dirac Hamiltonian, $H\Psi_{\tau,\tilde{\mu}} = \mathcal{E}_{\tau}\Psi_{\tau,\tilde{\mu}}$, are doublets ($\tilde{S} = 1/2$, $\tilde{\mu} = \pm 1/2$) with respect to the SU(2) generators \hat{S}_i

$$\hat{S}_z \Psi_{\tau,\tilde{\mu}} = \tilde{\mu} \Psi_{\tau,\tilde{\mu}}, \quad \hat{S}_{\pm} \Psi_{\tau,\tilde{\mu}} = \sqrt{(1/2 \mp \tilde{\mu})(3/2 \pm \tilde{\mu})} \Psi_{\tau,\tilde{\mu} \pm 1}, \quad (2)$$

where $\hat{S}_{\pm} = \hat{S}_x \pm i\hat{S}_y$. The eigenvalue τ refers to the other necessary quantum numbers.

However, the exact symmetry limit can not be realized in nuclei, because, if $V_S + V_V = \text{constant}$, there are no Dirac bound valence states and hence nuclei can not exist [22,16].

4. REALISTIC RELATIVISTIC MEAN FIELDS

A near equality in the magnitude of mean fields, $V_S \approx -V_V$, is a universal feature of the relativistic mean field approximation (RMF) of relativistic field theories with interacting nucleons and mesons [23] and relativistic theories with nucleons interacting via zero range interactions [24], as well as a consequence of QCD sum rules [25]. Recently realistic relativistic mean fields were shown to exhibit approximate pseudospin symmetry in both the energy spectra and wave functions [9,26,27]. In Table 1 pseudospin-orbit splittings calculated in the RMF [9] are compared with the measured values and are seen to be larger than the measured splittings which demonstrates that the pseudospin symmetry is better conserved experimentally than mean field theory would suggest. As mentioned in the last section pseudospin symmetry implies that the spatial wavefunction for the lower component of the Dirac wavefunctions will be equal in shape and magnitude for the two states in the doublet. In Figure 2, for example, the lower components of the $(2s_{1/2}, 1d_{3/2})$ Dirac eigenfunctions are approximately identical whereas the upper components have a different number of nodes [9]. We also note that the lower components of the Dirac wavefunction are much smaller than the upper components which is consistent with the view that nuclei are primarily non-relativistic. Nevertheless the understanding of the pseudospin symmetry involves relativity.

5. MAGNETIC DIPOLE AND GAMOW TELLER TRANSITIONS

Because the lower components are small, in order to test the pseudospin symmetry prediction that the lower components are almost identical we must observe transitions for which the upper components are not dominate. Magnetic dipole and Gamow-Teller transitions between the states in the doublet are forbidden non-relativistically since the orbital angular momentum of the two states differ by two units, but are allowed relativistically. Pseudospin symmetry predicts that, if the magnetic moments, μ , of the two states

Figure 2. The upper a) and lower components b) of $(2s_{1/2}, 1d_{3/2})$ Dirac eigenfunctions in ^{208}Pb [9].

are known, the magnetic dipole transition, $B(M1)$, between the states can be predicted. Likewise if the Gamow - Teller transitions between the states with the same quantum numbers are known, the transition between the states with different quantum numbers can be predicted [28]. For example for neutrons, the M1 transition is given by

$$\sqrt{B(M1 : n_r - 1, \ell + 2, j' \rightarrow \tilde{n}_r, \ell, j)}_\nu = -\sqrt{\frac{j+1}{2j+1}}(\mu_{j,\nu} - \mu_{A,\nu}), \quad (3)$$

$$\sqrt{B(M1 : n_r - 1, \ell + 2, j' \rightarrow \tilde{n}_r, \ell, j)}_\nu = \frac{j+2}{2j+3} \sqrt{\frac{2j+1}{j+1}}(\mu_{j',\nu} + \frac{j+1}{j+2}\mu_{A,\nu}), \quad (4)$$

where $j' = \ell + 3/2$, $j = \ell + 1/2$ and $\mu_{A,\nu} = -1.913\mu_0$ is the anomalous magnetic moment. A survey of forbidden magnetic dipole transitions taking into account the single-particle corrections by using spectroscopic factors shows a reasonable agreement with these relations, an example of which is given in Figure 3 [29].

6. PSEUDOSPIN PARTNERS

In the pseudospin symmetry limit, the eigenstates of the Dirac Hamiltonian in the doublet are connected by the pseudospin generators as given by (2). For nuclei, pseudospin symmetry is broken and the pseudospin partner produced by the raising and lowering operators on an eigenstate will not necessarily be an eigenstate. The question is: how different is the pseudospin partner from the eigenstate with the same quantum numbers? Energy splittings suggest that the symmetry breaking is small (Table 1) but is the breaking in the eigenfunctions small as well?

Figure 3. The experimental and theoretical B(M1) between one-particle or one-hole (circles) states and three-particle or three hole (squares) states in the doublet. The straight line denotes perfect agreement.

The pseudospin generators operating on the spherical basis which has pseudo-orbital angular momentum and spin coupled to total angular momentum j and total angular momentum projection m will give

$$S_q |n_r - 1, \ell + 2, j', m\rangle = A_{j',j'} |n_r - 1, \ell + 2, j', m'\rangle + A_{j',j} |n_r, \ell, j, m'\rangle_{psp}, \quad (5)$$

$$S_q |n_r, \ell, j, m\rangle = A_{j,j} |n_r, \ell, j, m'\rangle + A_{j,j'} |n_r - 1, \ell + 2, j', m'\rangle_{psp}, \quad (6)$$

where $A_{j,j'} = (-1)^{\frac{1}{2}-m'+\bar{\ell}} \sqrt{\frac{3(2j+1)(2j'+1)}{2}} \begin{pmatrix} j' & 1 & j \\ -m' & q & m \end{pmatrix} \left\{ \begin{matrix} \frac{1}{2} & \bar{\ell} & j \\ j' & 1 & \frac{1}{2} \end{matrix} \right\}$. In the pseudospin limit

$$|n_r, \ell, j, m'\rangle_{psp} = |n_r, \ell, j, m'\rangle, \quad V_S + V_V = \text{constant}, \quad (7)$$

$$|n_r - 1, \ell + 2, j', m'\rangle_{psp} = |n_r - 1, \ell + 2, j', m'\rangle, \quad V_S + V_V = \text{constant}. \quad (8)$$

Since the lower components of the eigenstates are operated on by the usual spin operator, \hat{s}_i , the radial function of the pseudospin partner is the same as the original eigenstate and hence Figure 2b shows how close the partner and eigenstate are. For the upper component, the operator $\hat{\tilde{s}}_i$ intertwines space and spin degrees of freedom through $U_p = 2\frac{\vec{s}\cdot\vec{p}}{p}$, thus $\hat{\tilde{s}}_i$ changes both the radial quantum number and the orbital angular momentum. In Figure 4 we compare the spatial wavefunctions of the upper components of the pseudospin partner with the eigenstate with the same quantum numbers and we see that the wavefunctions are almost equal in the interior but differ significantly as the surface is approached. These differences shall be discussed in more detail [30].

Figure 4. a) The $2s_{1/2}$ partner of $1d_{3/2}$ compared to the $2s_{1/2}$ eigenstate and b) the $1d_{3/2}$ partner of $2s_{1/2}$ compared to the $1d_{3/2}$ eigenstate for relativistic mean field eigenfunctions for ^{208}Pb [9].

7. QCD SUM RULES

Applying QCD sum rules in nuclear matter [25], the ratio of the scalar and vector self-energies were determined to be $\frac{V_S}{V_V} \approx -\frac{\sigma_N}{8m_q}$ where σ_N is the sigma term which arises from the spontaneous breaking of chiral symmetry [31]. For reasonable values of σ_N and quark masses, this ratio is close to -1. The implication of these results is that chiral symmetry breaking is responsible for the scalar field being approximately equal in magnitude to the vector field, thereby producing pseudospin symmetry.

8. NUCLEON - NUCLEUS SCATTERING

The relativistic optical model scalar and vector potentials determined from nucleon-nucleus scattering are almost equal and opposite in sign [32,33]. Since pseudospin symmetry

doesn't care if the potentials are complex, this symmetry may arise in nucleon-nucleus scattering [34]. The pseudospin and spin symmetry breaking can be determined empirically if the polarization, P , and spin rotation function, Q , are both measured as a function of the scattering angle [35]. The scattering amplitude, f , for the elastic scattering of a nucleon with momentum k on a spin zero target in the spin representation is [36],

$$f = A(k, \theta) + B(k, \theta) \boldsymbol{\sigma} \cdot \hat{\mathbf{n}}. \quad (9)$$

and in the pseudospin representation is

$$f = \tilde{A}(k, \theta) + \tilde{B}(k, \theta) \tilde{\boldsymbol{\sigma}} \cdot \hat{\mathbf{n}}. \quad (10)$$

The ratio, $\left|\frac{B}{A}\right|^2$, measures the amount of spin breaking and is given by [37],

$$\left|\frac{B}{A}\right|^2 = \frac{4}{2 + 2\sqrt{1 - P^2 - Q^2} - P^2 - Q^2} \left[\left(\frac{P}{2}\right)^2 + \left(\frac{Q}{2}\right)^2 \right] \quad (11)$$

whereas $\left|\frac{\tilde{B}}{\tilde{A}}\right|^2$ measures the amount of pseudospin breaking,

$$\left|\frac{\tilde{B}}{\tilde{A}}\right|^2 = \frac{\tan^2(\theta) - Q \tan(\theta) + \left|\frac{B}{A}\right|^2 (1 - Q \tan(\theta))}{1 + Q \tan(\theta) + \left|\frac{B}{A}\right|^2 (\tan^2(\theta) + Q \tan(\theta))}. \quad (12)$$

In Figure 5 the square of the ratio of the pseudospin dependent amplitude to the pseudospin independent amplitude, $(\tilde{B}/\tilde{A})^2$, and the square of the ratio of the spin dependent amplitude to the spin independent amplitude, $(B/A)^2$, are plotted [35,37] for 800 MeV proton scattering [38] on ^{208}Pb . The pseudospin spin breaking is at most of the order of 10%, a factor of three lower than the spin breaking. On the other hand low energy proton scattering indicates that pseudospin is badly broken [34].

9. SUMMARY

We have shown that pseudospin symmetry is an SU(2) symmetry of the Dirac Hamiltonian with mean fields, $V_S = -V_V + \text{constant}$. This symmetry predicts that the eigenfunctions of the Dirac Hamiltonian will be pseudospin doublets connected by the SU(2) generators. In nuclei, pseudospin symmetry must be broken since nuclei are not bound in this limit. However relativistic mean field approximations of relativistic nuclear field theories and relativistic nuclear Lagrangians with zero range interactions give scalar and vector mean field potentials in nuclei, $V_S \approx -V_V$. We have shown that the pseudospin partners of the eigenstates of these realistic relativistic Dirac Hamiltonians are themselves approximate eigenstates of the same Dirac Hamiltonian. Furthermore, since the realistic relativistic mean field calculations overestimate the energy splitting between doublets compared to their measured values, these same calculations may overestimate the difference between the experimental eigenfunctions.

Another consequence of pseudospin symmetry is that, if the magnetic moments of the states in the doublet are known, the “ ℓ ” forbidden magnetic dipole transitions can be predicted. Similar relationships for Gamow-Teller transitions between states in the doublet hold as well.

Figure 5. The spin (dots), $\left|\frac{B}{A}\right|^2$, and pseudospin (squares), $\left|\frac{\tilde{B}}{A}\right|^2$, breaking probabilities are plotted versus the scattering angle [35,37] for 800 MeV proton scattering [38] on ^{208}Pb .

Pseudospin symmetry has been shown to be approximately conserved in medium energy nucleon-nucleus scattering from nuclei.

In the future these issues will be investigated in deformed nuclei as well.

Finally, pseudospin symmetry has been linked via QCD sum rules to chiral symmetry breaking in nuclei. This suggests a more fundamental significance which needs to be explored. Recently much effort has been expended in deriving effective field theories based on QCD and chiral perturbation theory [39]. However these effective field theories are generally non-relativistic field theories. Perhaps the lesson of pseudospin symmetry is that relativistic effective theories should be derived as well, at least as an intermediate step towards non-relativistic field theories, in order to understand qualitative aspects about the physics of nuclei which may be missed by non-relativistic theories.

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(keV)

(9/2⁻) 508.22

(7/2⁻) 333.26

5/2⁻ 187.40

3/2⁻ 74.33

1/2⁻ 0
1/2[510]



$\tilde{\Lambda} = 1$

(keV)

(11/2⁻) 511.6

(9/2⁻) 341.5

7/2⁻ 190.60

5/2⁻ 75.04

3/2⁻ 9.746
3/2[512]









